Deadline: October 7, 5pm. Please upload your assignment to Moodle by this time, and also bring a paper copy to class on next day. This assignment will be graded anonymously, so please don't list your name, but only your MAC ID.

As noted by the syllabus as well as in class, the scope, content, and convention of assignments are set by lectures, instead of any specific textbook. Please beware that different textbooks may use different symbolism or definitions.

Assignments are meant to be challenging! You are encouraged to discuss your answers with other students (but write up your own answers individually).

1. Find all the pairs of sentences in Assignment 3.1 that are tautologically equivalent. Fill the following table, by writing '+' in every square for which the row sentence is equivalent to the column sentence.

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

For each pair without a '+', show that there is a truth-value assignment to the sentential variables, under which the two sentences get different values.

Note: You can put '+' in the diagonal and you can also assume that the filled table is symmetric around the diagonal. (Can you see why?) This leaves fifteen pairs of sentential expressions for checking. Since equivalent sentences have always the same truth-values, they behave in the same way with respect to other sentences. Hence, the more equivalent pairs you discover at an early stage, the more you will economize in checking.

2. Recall what Modus Tollens (or the Tail Argument) is. Use the equivalence relations for conditionals (see below) to explain why Modus Tollens is valid, and why, for a true conditional, the consequent is a necessary condition for the antecedent.

$$A \rightarrow B \equiv \neg A \vee B \qquad A \vee B \equiv \neg A \rightarrow B$$
$$A \rightarrow B \equiv \neg(A \wedge \neg B) \qquad A \wedge B \equiv \neg(A \rightarrow \neg B)$$

3. Find all the tautologies and all the contradictions among the following sentences. For sentences not listed as a tautologies (or as contradictions), give a truth-value assignment to the sentential variables under which the sentence gets F (or gets T).

1. $\neg(A \vee B) \vee (A \vee B)$
2. $A \wedge (\neg(A \vee B) \vee (C \wedge \neg A))$
3. $(A \wedge B) \vee (\neg A \wedge \neg B)$
4. $(A \vee B) \wedge (\neg A \vee \neg B)$

4. Each of the following is an instance of one of the basic equivalence laws we learned in class on September 26. Find the law and the substitution that has been used to get the instance.

Note: Sometimes the two sides of the equivalence have been switched around.

1. $\neg(\neg A \wedge B \vee \neg A) \;\equiv\; \neg(\neg A \wedge B) \wedge \neg\neg A$
2. $\neg(A \vee \neg B) \wedge \neg(B \wedge C) \;\equiv\; \neg((A \vee \neg B) \vee (B \wedge C))$
3. $\neg(\neg(A \vee B) \wedge B) \;\equiv\; \neg\neg(A \vee B) \vee \neg B$
4. $\neg B \vee (C \wedge \neg A) \;\equiv\; (\neg B \vee C) \wedge (\neg B \vee \neg A)$
5. $(\neg(A \vee B) \wedge A) \vee (\neg(A \vee B) \wedge A) \;\equiv\; \neg(A \vee B) \wedge (A \vee A)$
6. $A \wedge (B \vee C) \vee A \wedge C \;\equiv\; A \wedge C \vee A \wedge (B \vee C)$
7. $(B \vee A) \wedge ((A \vee B) \wedge (B \vee A)) \;\equiv\; ((B \vee A) \wedge (A \vee B)) \wedge (B \vee A)$
8. $\neg(C \wedge (A \vee C)) \;\equiv\; \neg C \vee \neg(A \vee C)$

Here're some of the equivalence laws:

Double Negation	$\neg\neg A \equiv A$
Commutativity	$A \wedge B \equiv B \wedge A$ $A \vee B \equiv B \vee A$
Associativity	$(A \wedge B) \wedge C \equiv A \wedge (B \wedge C)$ $(A \vee B) \vee C \equiv A \vee (B \vee C)$
Idempotence	$A \wedge A \equiv A$ $A \vee A \equiv A$
Distributivity	$A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$ $A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C)$
De Morgan's Laws	$\neg(A \wedge B) \equiv \neg A \vee \neg B$ $\neg(A \vee B) \equiv \neg A \wedge \neg B$

5. Express the following texts as sentential expressions using sentential variables and connectives that we have learned (that is, negation, conjunction, disjunction, conditional, and biconditional). Also explain what each sentential variable represents; for example, A: Alex is happy. Indicate relevant ambiguities, as you find them, and formalize each of the readings.
   - If the lines go down, then the transformer blows and the power goes out.
   - The power goes out if the lines go down or the transformer blows.
   - You'll get well in the world if you are neither more nor less wise, neither better nor worse than your neighbors.
   - There is time enough for everything in the course of the day if you do but one thing at once; but there is not time enough in the course of the day if you will do two things at a time.
   - If the mind, which rules the body, ever forgets itself so far as to trample upon its slave, the slave is never generous enough to forgive the injury; but will rise and smite its oppressor.