Deadline: October 28, 5pm. Please upload your assignment to Moodle by this time, and also bring a paper copy to class on next day. This assignment will be graded anonymously, so please don't list your name, but only your MAC ID.

As noted by the syllabus as well as in class, the scope, content, and convention of assignments are set by lectures, instead of any specific textbook. Please beware that different textbooks may use different symbolism or definitions.

This assignment is less challenging than normal. It's meant to guide you to understand logical implication. But you are still encouraged to discuss your answers with preceptors and other students (but write up your own answers individually).

1. What is the definition of tautological implication? How is it related to logical implication? How is it related to tautology (conceptually)?

2. Based on the answers to the last question, explain why a tautology is logically implied by anything and why a contradiction logically implies anything.

3. How do tautological (or logical) implications differ from conditionals?

   Please also explain their differences by explaining why $\boldsymbol{A} \models \boldsymbol{B}$ if and only if $\models \boldsymbol{A} \rightarrow \boldsymbol{B}$.

4. A logical implication does NOT hold if it is possible to assign the sentential variables truth-values under which the left-hand side of $\models$ gets **T** and the right-hand side gets **F**. Thus, we just need to identify such an assignment in order to show an implication does not hold. For example, in the case of $A \rightarrow B \models \neg A$,

   - Assume that B is true. From the truth table of conditionals, we know that $A \rightarrow B$ gets T, regardless whether A is true or false. If A gets T, then $\neg A$ gets F.

     Therefore, the implication does not hold.

   A logical implication does hold if it is impossible to assign the sentential variables truth-values under which the left-hand side of $\models$ gets **T** and the right-hand side gets **F**. Thus, to show that an implication holds, we need check all possible assignments and see in every case in which the left-hand side of $\models$ gets **T**, the right-hand side also gets **T**. For example, in the case of $\neg A \models A \rightarrow B$

   (i) Assume that B is false. If $\neg A$ gets T, then A must get F. In that case, from the truth table of conditionals, we know that $A \rightarrow B$ gets T;

(ii) Assume that B is true. If $\neg A$ gets T, then A must get F. In that case, from the truth table of conditionals, we know that $A \to B$ gets T.

Therefore, there is no assignment (to the sentential variables) in which $\neg A$ gets T and $A \to B$ doesn't.

Find, for each of (1) $A \models A \vee B$, (2) $B \models A \to B$, (3) $A \wedge B \models A \leftrightarrow B$, whether the reverse implication (switch the left and right-hand sides) holds for all B, in each of the following cases:

(I) A is true. (II) A is false.

Prove every positive answer by an argument of the type given above. Prove every negative answer by a suitable counterexample.

5. Recall Conditional Proof: $\boldsymbol{\Gamma \models A \to B}$ if and only if $\boldsymbol{\Gamma, A \models B}$. Using the definition of tautological implication and the truth table of conditionals, explain Conditional Proof as we did in class (and above), that is, explain why

   (1) assume $\boldsymbol{\Gamma \models A \to B}$, then $\boldsymbol{\Gamma, A \models B}$;

   (2) assume $\boldsymbol{\Gamma, A \models B}$, then $\boldsymbol{\Gamma \models A \to B}$.

6. (Optional) Using the "distributive laws" of Assignment 5.5, push → all the way in, in the following sentences:

   (1) $(A \vee B) \to (C \vee D)$

   (2) $(A \vee B) \to (C \wedge D)$

   (3) $(A \wedge B) \to (C \vee D)$

   (4) $(A \wedge B) \to (C \wedge D)$

   Note: Using conditional, we can form the tautology $A \to A$, which is perhaps simpler than our previous standard tautology $A \vee \neg A$.