

Deadline: Nov. 4, 5pm. Please upload your assignment to Moodle by this time, and also bring a paper copy to class on next day. This assignment will be graded anonymously, so please don't list your name, but only your MAC ID.

As noted by the syllabus as well as in class, the scope, content, and convention of assignments are set by lectures, instead of any specific textbook. Please beware that different textbooks may use different symbolism or definitions.

Assignments are meant to be challenging! You are encouraged to discuss your answers with other students (but write up your own answers individually).

- Using the implication laws introduced so far, prove, via top-down derivations, the following five implications. The goal should be reduced in the end to an obvious implication in which the conclusion is one of the premises. You can use substitution-of-equivalents based on simple equivalences as we did in class. In the derivations of (4) and (5), you can use the implications $B \models A \vee B$ and $A, B \models A \wedge B$.

$$(1) \quad \models [A \rightarrow (B \rightarrow C)] \rightarrow [(A \rightarrow B) \rightarrow (A \rightarrow C)]$$

$$(2) \quad \neg A \rightarrow B, B \rightarrow C \models \neg C \rightarrow A$$

$$(3) \quad A \rightarrow (B \vee C), \neg B \models A \rightarrow C$$

$$(4) \quad (A \vee B) \rightarrow (B \rightarrow C) \models B \rightarrow C$$

$$(5) \quad (A \wedge B) \rightarrow C, B \models A \rightarrow C$$

- Show, via top-down derivations, that the following implications obtain. The final goals should be of the two allowed kinds of self-evident implications.

$$(1) \quad \models (A \rightarrow \neg A) \rightarrow \neg A$$

$$(2) \quad \models (A \rightarrow B) \rightarrow (\neg B \rightarrow \neg A)$$

$$(3) \quad (A \vee B) \rightarrow C \models (A \rightarrow C) \wedge (B \rightarrow C)$$

$$(4) \quad \models [(A \rightarrow B) \wedge (\neg A \rightarrow B)] \rightarrow B$$

$$(5) \quad A \rightarrow (B \wedge C), (B \rightarrow D) \vee (C \rightarrow D) \models A \rightarrow C$$

$$(6) \quad A \rightarrow B, B \rightarrow \neg B \models \neg A$$

$$(7) \quad A \rightarrow (B \wedge C), B \rightarrow \neg C \models \neg A$$

$$(8) \quad (A \wedge B) \rightarrow C, (A \wedge \neg B) \rightarrow C \models A \rightarrow C$$